1 Problem 1:

Calculate the **Present Value**, to the nearest dollar, for each of the following cash flows. Assume a **10% interest rate** for each. Show your work. (The relevant formulas are in chapter 4 of Mishkin)

1.1 1100 dollars next year, 1210 dollars two years from now, and 1331 dollars three years from now.

$$PV = \frac{1100}{(1.10)^1} + \frac{1210}{(1.10)^2} + \frac{1331}{(1.10)^3}$$

$$PV = 1,000 + 1,000 + 1,000 = \mathbf{3,000}$$

1.2 7400 dollars, 21 years from now.

$$PV = \frac{7400}{(1.10)^{21}} = \frac{7400}{7.40024994} \approx \mathbf{1000}$$

(Note: I chose the numbers to make things very convenient. $(1.10)^{21} \approx 7.400$)

1.3 2000 dollars every year, forever.

Formula for perpetuity:

$$PV = \frac{C}{r} = \frac{2000}{0.10} = \mathbf{20,000}$$

You can find this formula in the book or derive it yourself, or you could use a spreadsheet and just add of the present value of future cash flows until things diverge.

1.4 2000 dollars every year, for 7 years

To the nearest dollar, present value is **$9737**.

Easiest way to find this is directly:

$$\begin{aligned} PV &= \frac{2000}{(1.10)^1} + \frac{2000}{(1.10)^2} + \frac{2000}{(1.10)^3} + \frac{2000}{(1.10)^4} + \frac{2000}{(1.10)^5} + \frac{2000}{(1.10)^6} + \frac{2000}{(1.10)^7} \\ &= 2000 \times \left(\frac{1}{1.10} + \frac{1}{1.10^2} + \frac{1}{1.10^3} + \frac{1}{1.10^4} + \frac{1}{1.10^5} + \frac{1}{1.10^6} + \frac{1}{1.10^7} \right) \\ &\approx 2000 \times 4.8684 \\ &\approx 9736.8 \end{aligned}$$

Note: If this were a perpetuity, PV would be 20k (see 1.3). So we get nearly half the value in the first 7 years.

2 Problem 2:

Calculate the **Yield to Maturity** for each of the following assets. Show your work, and verify that the present value of future payments equals the initial price you paid.

2.1 You pay 1000 dollars today and receive 1340 dollars six years from now.

YTM is the value of i that solves

$$1000 = \frac{1340}{(1+i)^6}$$

Solving:

$$i = \left(\frac{1340}{1000}\right)^{1/6} - 1 \approx 5\%$$

2.2 You pay 1000 dollars today. You receive 100 dollars every year for the next nine years. Then ten years from now, you receive 1100 dollars.

YTM is the value of i that solves

$$1000 = \frac{100}{(1+i)} + \frac{100}{(1+i)^2} + \frac{100}{(1+i)^3} + ... + \frac{1100}{(1+i)^{10}}$$

or equivalently

$$1000 = \left(\frac{100}{(1+i)} + \frac{100}{(1+i)^2} + \frac{100}{(1+i)^3} + ... + \frac{100}{(1+i)^{10}}\right) + \frac{1000}{(1+i)^{10}}$$

You could work it out algebraically (very difficult), guess and check using a spreadsheet, or you could recognize that this is a bond where the face value is the same as the price, and thus the book says the yield to maturity will simply be the coupon divided by the price. $YTM = \frac{100}{1000} = 10\%$

Regardless, you should plug this value of i in and crunch the numbers to make sure it works.

3 Problem 3:

This goes a bit beyond what's in the book. Suppose you have a fixed payment loan that pays C dollars each year for 30 years. If the interest rate is i, then the present value of the cash flow from this loan is :

$$PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + ... + \frac{C}{(1+i)^{30}}$$

With a bit of algebra, we can derive a simpler form:

$$PV = \frac{C}{i} \cdot \left(1 - \frac{1}{(1+i)^{30}}\right)$$

3.1 Algebraically derive the second formula from the first.

$$PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + ... + \frac{C}{(1+i)^{30}}$$

$$\begin{aligned} (1+i)PV &= C + \left(\frac{C}{(1+i)} + ... + \frac{C}{(1+i)^{29}}\right) \\ &= C + \left(PV - \frac{C}{(1+i)^{30}}\right) \end{aligned}$$

$$(1+i)PV - PV = C - \frac{C}{(1+i)^{30}}$$

$$i \cdot PV = C - \frac{C}{(1+i)^{30}}$$

$$PV = \frac{\left(C - \frac{C}{(1+i)^{30}}\right)}{i} = \frac{C}{i} \cdot \left(1 - \frac{1}{(1+i)^{30}}\right)$$

3.2 How much money could you borrow using a fixed payment loan with 30 annual fixed payments of 10,000 dollars each, and an interest rate of 5%? Show your work.

Easiest way to do this is plug in the formula from 3.1:

$$PV = \frac{C}{i} \cdot \left(1 - \frac{1}{(1+i)^{30}}\right)$$

$$\begin{aligned} PV &= \frac{10000}{0.05} \cdot \left(1 - \frac{1}{(1+0.05)^{30}}\right) \\ &\approx 200000 \cdot (1 - 0.231377) \\ &\approx 153725 \end{aligned}$$

The present value is the price the bank pays you for this loan - ie, the amount you borrow.

So you can borrow about **154 thousand dollars.**

Note: For actual mortgages and car loans, the bank will typically bundle your property tax and home insurance into one monthly payment alongside the payments on your mortgage. This formula is only applicable to the mortage part of that monthly payment.

4 Problem 4:

Suppose a Discount Bond promises 2000 dollars in one year's time. For each of the following prices for this bond, calculate the interest rate (yield to maturity): 2000 dollars, 1900 dollars, 1800 dollars, 1700 dollars, 1600 dollars. Show your work.

The YTM is the value of i that solves

$$Price = \frac{2000}{1+i}$$

Solving yields:

$$i = \frac{2000}{Price} - 1$$

or equivalently

$$i = \frac{2000 - Price}{Price}$$

Plugging these prices in:

Price	Interest Rate	Calculation
\$2000	0%	$i = \frac{2000-2000}{2000} = 0$
\$1900	10.5%	$i = \frac{2000-1900}{1900} = \frac{100}{1900} = \frac{1}{19} \approx 0.105$
\$1800	11.1%	$i = \frac{2000-1800}{1800} = \frac{200}{1800} = \frac{1}{9} = 0.11\bar{1}$
\$1700	17.6%	$i = \frac{2000-1700}{1700} = \frac{300}{1700} = \frac{3}{17} \approx 0.176$
\$1600	25%	$i = \frac{2000-1600}{1600} = \frac{400}{1600} = \frac{1}{4} = 0.25$

5 Problem 5:

Suppose the state of South Dakota reduces state taxes without reducing state spending, and has a state-level budget deficit as a result.

In about 20-30 words, describe what this might do to the supply and demand for South Dakota Municipal bonds, and why.

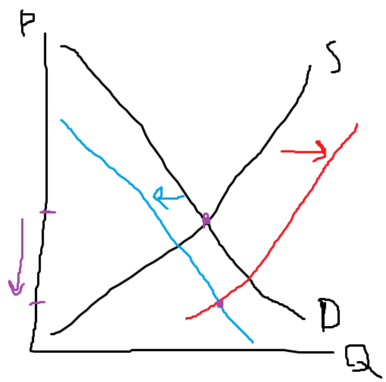
Attach a sketch of this shock to your homework, ala the depictions of shocks in Mishkin ch5. (I would prefer that you insert it into your submission file, if possible.) What do you expect would happen to equilibrium Price, Interest Rate, and Quantity sold for this bond?

The story I'm looking for is:

- Supply increases because SD needs to borrow to cover its deficit.
- Demand decreases because people worry about SD's ability to repay in the future.

One or both of these shocks is fine. Any explanation is fine. Don't deduct points for going over the word limit.

Here is a deliberately sloppy graph for the case where S increases and D decreases. Any graph is fine - hand-drawn, doodled, whatever, as long as it matches the story they tell above.



And regarding the change in equilibrium:

- If you said that Supply Increases and Demand decreases
  - Price goes down, interest rates go up, and the change in quantity is technically ambiguous.
- If you said only that Supply Increases
  - Price goes down, interest rates go up, and Quantity increases.
- If you said only Demand decreases
  - Price goes down, interest rates go up, and Quantity decreases.