

Firm has zero marginal cost and no competition. Owner has discount factor β , and sets a price at the beginning of time to maximize the sum of time-discounted revenue

Quantity demanded per period is equal to $q(p) * s_t$ where s_t is number of consumers aware of the firm's services, and $q(\cdot)$ is the fraction of aware consumers willing to buy the service at a given price.

If s_t exogenous, then the owner just ignores it and maximizes $pq(p)$.

If the firm can spend on marketing to increase s , $s_{t+1} = s_t + f(m)$, then they still choose p to maximize $pq(p)$. It just adds an investment problem.

But if consumers who purchase the product tell their friends about it, such that each purchase also increases s by μ , $s_{t+1} = s_t + \mu q(p_t)s_t$, then it would make sense to have a price that's 'too low'.

Owner wants to set price to maximize $\sum_{t=0}^{\infty} \beta^t \cdot q(p) \cdot p \cdot s_t = \sum_{t=0}^{\infty} \beta^t \cdot q(p) \cdot p \cdot (1 + \mu q(p))^t \cdot s_0$.

Assuming that $\max_p(1 + \mu q(p)) < \frac{1}{\beta}$, this is same as choosing p to maximize

$$\frac{q(p)p}{1 - (1 + \mu q(p))\beta}$$

Simple example: Let $q(p) = 1 - p$. Then without buyers telling their friends, optimal price is $p = \frac{1}{2}$.

If $\beta = \frac{9}{10}$ and $\mu = \frac{1}{10}$, where every ten actual buyers creates one additional potential buyer through word of mouth, then optimal price is $\frac{\sqrt{10}-1}{9} \approx 0.24$